INHARMONIC OVERTONES IN PARTIALLY PLATED AT-CUT QUARTZ CRSTAL PLATES

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Spurious modes caused by inharmonic overtones are one of the most familiar problems of AT-cut quartz resonators. Mindlin and Lee had explained this phenomenon theoretically in detail in the 1960's[1]. Their calculation results were also shown to be in good agreement with the experimental data from Curran and Koneval, which are based on quartz plates with relatively large length, width and thickness. Today, the dimensions of quartz plates have decreased enormously due to continuous miniaturization. Whether the calculations of Mindlin and Lee's theory still have great agreement with the measurement data of the miniaturization AT-cut quartz plate is the problem we have to solve. In this paper, the frequency spectrum of quartz plates with partial electrodes at different dimensions was calculated and compared with the experimental data of miniaturized plates. The influence of the length and width of quartz crystal plates and electrodes, and size of mounting area are also studied. It shows the theory still can estimate frequency of inharmonic overtone quite well and the dimensions of plates have little influence on inharmonic overtones. Such results are useful in the design improvement process of quartz crystal resonators.

Keywords: Inharmonic overtone; AT-cut; Quartz resonator; Miniaturization; Vibration

1. INTRODUCTION

Due to highly anisotropic properties of quartz, the vibrations of quartz plate are very complex. Mindlin did series work in crystal plate vibration, as the base of AT-cut quartz plate vibration analysis. Mindlin, et al. [1-3] presented the thickness-shear and flexural vibrations of crystal plates under free, forced, and contoured shape conditions; and then [4, 5] considered the piezoelectric effects and incomplete electrode to solve the rectangular plate problem. Mindlin and Lee [6] studied the thickness-shear, inharmonic overtones and flexural vibrations of partially plated crystal plate, which was infinite along diagonal axis, and explained Bechmann's Number and the effect of size of electrodes on Q very well. Wang and Zhao [7] presented the dispersion relations of thickness-shear, flexural and extensional vibration and the mode charts to determine the optimal length of crystal blanks which was finite dimension but without electrodes.

Today, the dimension of the smallest commercialized AT-cut quartz crystal chip is about 1.0mm by 0.8mm. The electrode area should not be too small, since the resistance will be too high to work. For miniaturized AT-cut quartz chip, the real layout of the crystal plate and electrode is quite different form the infinite dimension hypothesis. At the same time, the electrode effect is very important and should not be neglected. However, it is inefficiency and expensive if a crystal engineer design the resonator only by the commercial finite element analysis tool. In this paper, three vibration modes, thickness-shear, inharmonic overtone and flexural, are considered by Mindlin and Lee's theory. The mode chart and displacements of the finite dimension crystal plate with strip electrode are calculated, and the corresponding experiment data are measured. The results show that the simulation tool developed by Mindlin and Lee's theory could provide a crystal engineer much useful information for small size resonator design.

2. THE FIRST ORDER MINDLIN PLATE EQUATIONS

Consider an AT-cut rectangular plate of thickness *2b*,

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length *2l*, which is partially plated by an electrode of length *2a*, as shown in Fig. 1. Following Ref. [6] we only take displacements $u_1^{(1)}$ and $u_2^{(0)}$ into account for thickness shear, flexure and inharmonic overtones. Then, with the assumption of straight-crested waves, set the displacements for plated portion $-a \leqq x_1 \leqq a$ as:

$$\bar{u}_2^{(0)} = A_1 \sin \bar{\xi} x_1 e^{iwt}, \quad \bar{u}_1^{(1)} = \frac{A_2}{b} \cos \bar{\xi} x_1 e^{iwt}. \tag{1}$$

For unplated portion $a \leqq x_1 \leqq l$, we take

$$u_2^{(0)} = \left[A_3 \sin(\xi x_1 - \xi a) + A_4 \cos(\xi x_1 - \xi a)\right] e^{iwt}, \quad u_1^{(1)} = \left[\frac{A_5}{b}\cos(\xi x_1 - \xi a) + \frac{A_6}{b}\sin(\xi x_1 - \xi a)\right] e^{iwt}, \tag{2}$$

where $A_i\,(i = 1,2,3,4,5,6); \xi, \omega$ and t are vibration amplitudes, wavenumber, frequency, and time, respectively.

Stress-displacement relations for plated and unplated portions are:

$$\bar{T}_6^{(0)} = 2\bar{\kappa}_6^2 b c_{66} (\bar{u}_{2,1}^{(0)} + \bar{u}_1^{(1)}), \quad \bar{T}_1^{(1)} = \frac{2b^3}{3} \tilde{c}_{11} \bar{u}_{1,1}^{(1)}; \tag{3}$$

$$T_6^{(0)} = 2\kappa_6^2 b c_{66} (u_{2,1}^{(0)} + u_1^{(1)}), \quad T_1^{(1)} = \frac{2b^3}{3} \tilde{c}_{11} u_{1,1}^{(1)}. \tag{4}$$

The stress equations of motion are:

$$\bar{T}_{6,1}^{(0)} = 2b\rho(1+R)\ddot{\bar{u}}_2^{(0)}, \quad \bar{T}_{1,1}^{(1)} - \bar{T}_6^{(0)} = \frac{2b^3}{3}\rho(1+3R)\ddot{\bar{u}}_1^{(1)}. \tag{5}$$

$$T_{6,1}^{(0)} = 2b\rho \ddot{u}_2^{(0)}, \quad T_{1,1}^{(1)} - T_6^{(0)} = \frac{2b^3}{3}\rho \ddot{u}_1^{(1)}. \tag{6}$$

By substituting Eqs. (1) and (2) into Eqs. (5) and (6), respectively, the displacement-equations of motion are

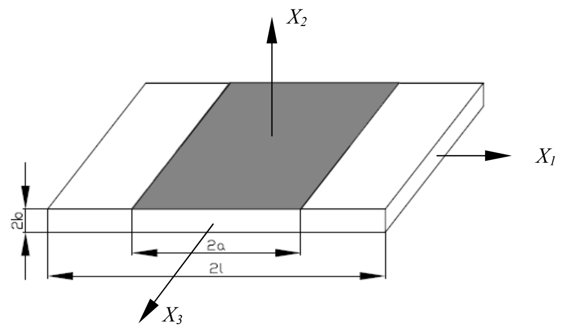

Figure 1. Rectangular AT-cut quartz plate with an electrode strip.

obtained:

$$\begin{bmatrix} -\bar{\kappa}_6^2 \bar{Z}^2 + (1+R)\Omega^2 & -\frac{2\bar{\kappa}_6^2}{\pi}\bar{Z} \\ -6\bar{\kappa}_6^2 \pi \bar{Z} & -\frac{\tilde{c}_{11}}{c_{66}}\pi^2 \bar{Z}^2 - 12\bar{\kappa}_6^2 + (1+3R)\pi^2\Omega^2 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = 0; \tag{7}$$

$$\begin{bmatrix} 12\Omega^2 - \pi^2 Z^2 & -2\pi Z \\ -\frac{\pi Z}{2} & -\frac{\tilde{c}_{11}}{c_{66}} Z^2 + \Omega^2 - 1 \end{bmatrix} \begin{bmatrix} A_3 \\ A_5 \end{bmatrix} = 0; \tag{8}$$

$$\begin{bmatrix} 12\Omega^2 - \pi^2 Z^2 & 2\pi Z \\ \frac{\pi Z}{2} & -\frac{\tilde{c}_{11}}{c_{66}} Z^2 + \Omega^2 - 1 \end{bmatrix} \begin{bmatrix} A_4 \\ A_6 \end{bmatrix} = 0; \tag{9}$$

with normalized frequency and wavenumber defined as:

$$\Omega = \frac{\omega}{\omega_0}, Z = \frac{\xi}{\frac{\pi}{2b}}, \bar{Z} = \frac{\bar{\xi}}{\frac{\pi}{2b}}, \omega_0 = \frac{\pi}{2b}\sqrt{\frac{c_{66}}{\rho}}$$

Let the coefficient determinant of Eqs. (7), (8), (9), vanish, we get the ratios of amplitudes as:

$$\alpha_{1r} = \frac{A_{1r}}{A_{2r}} = -\frac{-\frac{2\bar{\kappa}_6^2}{\pi}\bar{Z}_r}{-\bar{\kappa}_6^2 \bar{Z}^2 + (1+R)\Omega^2}, r = 1,2, \tag{10}$$

$$\alpha_{3r} = \frac{A_{3r}}{A_{5r}} = -\frac{-2\pi Z_r}{12\Omega^2 - \pi^2 Z_r^2}, r = 1,2, \tag{11}$$

$$\alpha_{4r} = \frac{A_{4r}}{A_{6r}} = -\frac{2\pi Z_r}{12\Omega^2 - \pi^2 Z_r^2}, r = 1,2, \tag{12}$$

Consequently, displacements can be written in this form:

$$\bar{u}_2^{(0)} = \left[A_{21}\alpha_{11}\sin(\frac{\pi\bar{Z}_1}{2}\frac{x_1}{b}) + A_{22}\alpha_{12}\sin(\frac{\pi\bar{Z}_2}{2}\frac{x_1}{b})\right]e^{iwt},$$
$$\bar{u}_1^{(1)} = \left[\frac{A_{21}}{b}\cos(\frac{\pi\bar{Z}_1}{2}\frac{x_1}{b}) + \frac{A_{22}}{b}\cos(\frac{\pi\bar{Z}_2}{2}\frac{x_1}{b})\right]e^{iwt}. \quad (13)$$

$$u_2^{(0)} = \left\{\begin{array}{l} A_{51}\alpha_{31}\sin\left[\frac{\pi Z_1}{2}\frac{(x_1-a)}{b}\right] + A_{52}\alpha_{32}\sin\left[\frac{\pi Z_2}{2}\frac{(x_1-a)}{b}\right] \\ + A_{61}\alpha_{41}\cos\left[\frac{\pi Z_3}{2}\frac{(x_1-a)}{b}\right] + A_{62}\alpha_{42}\cos\left[\frac{\pi Z_4}{2}\frac{(x_1-a)}{b}\right] \end{array}\right\}e^{iwt},$$
$$u_1^{(1)} = \left\{\begin{array}{l} \frac{A_{51}}{b}\cos\left[\frac{\pi Z_1}{2}\frac{(x_1-a)}{b}\right] + \frac{A_{52}}{b}\cos\left[\frac{\pi Z_2}{2}\frac{(x_1-a)}{b}\right] \\ + \frac{A_{61}}{b}\sin\left[\frac{\pi Z_3}{2}\frac{(x_1-a)}{b}\right] + \frac{A_{62}}{b}\sin\left[\frac{\pi Z_4}{2}\frac{(x_1-a)}{b}\right] \end{array}\right\}e^{iwt}, \quad (14)$$

For the assumption that the edges of the plates are free, the stresses require:

$$T_6^{(0)} = T_1^{(0)} = 0, \text{at } x_1 = \pm l. \quad (15)$$

And for the conditions of continuity at the boundaries of the plated and unplated portions of the plate, displacements and stresses require:

$$\begin{array}{l} u_2^{(0)} = \bar{u}_2^{(0)}, u_1^{(1)} = \bar{u}_1^{(1)}, \\ T_6^{(0)} = \bar{T}_6^{(0)}, T_1^{(1)} = \bar{T}_1^{(1)}, \end{array} \text{ at } x_1 = \pm a \quad (16)$$

Then, by substituting Eqs. (13) and (14) into the boundary conditions, the spectra of frequency versus blank length to thickness ratio *l/b* and frequency versus electrode length to thickness ratio *a/b* are computed, as shown in Fig.2 and 3.

3. EXPERIMENTAL RESULTS

Eqs. (13), (14), (15) and (16), provide frequency as a function of *l/b*, *a/b* and *R*. By this function we have calculated the frequency spectra about length of blank and electrode, as shown in Figs. 2 and 3. Intended to know the vibration shapes of the frequencies marked on Fig. 3, the displacements are calculated by substituting amplitude ratios to Eqs. (13) and (14).According to Figs. 4, 5 and 6, we find the modes are thickness shear, first inharmonic overtone and flexure respectively.

Aiming at researching the characteristics of miniaturization quartz crystal resonators, we choose a kind of SMD type seam crystal resonators as the subject of experiments, whose package size is 3.2mm*2.5mm *08mm and frequency of thickness shear is 40MHz. By testing different lengths and widths of blanks and electrodes, we get the measurement data as shown in Figs. 2, 3 and Table 1.

From Fig. 2, we can see that the measured frequency of thickness shear and first inharmonic overtone matching quite well with the frequency spectra. And the change of length has little influence to these two modes. Otherwise, from Fig. 3, the length of electrode has remarkable relationship with these two modes. It is clear that the bigger the length, the lower the frequency. And the result of calculation also agrees with the measurement data well.

Blanks with the same length of electrode and different widths have quite close frequencies of thickness shear and inharmonic overtone as shown in Fig.3. On the other words, the influence of the electrode's width is not remarkable.

Table 1 gives the frequencies of thickness shear and inharmonic overtone of different width of quartz plate. From the table we can also detect that the width of blank has little influence to these two modes.

The experiments about the mounting sizes also have been done. But the effect of mounting sizes is not evident.

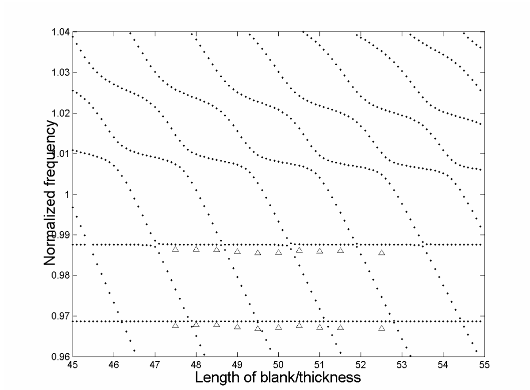

Figure 2. Calculated resonant frequencies of thickness shear, inharmonic overtones and flexure, in partially plated AT-cut quartz plate for *R*=0.035, *a/b*=22(●), and measurement data of quartz crystal resonators for *R*=0.035, *a/b*=22, width of blank/thickness=33, width of electrode/thickness=14 (△).

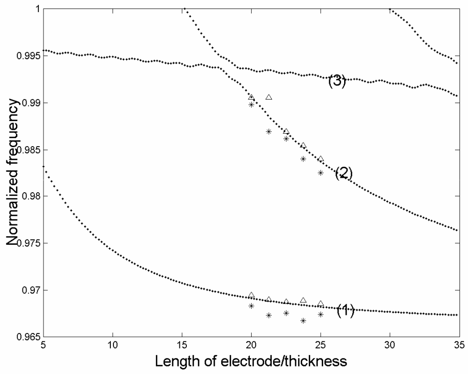

Figure 3. Calculated resonant frequencies of thickness shear, inharmonic overtones and flexure, in partially plated AT-cut quartz plate for *R*=0.035, *l*/*b*=50(●), and measurement data of quartz crystal resonators for *R*=0.035, *l*/*b*=50, width of blank/thickness=33, width of electrode/thickness=14 (△),22(∗).

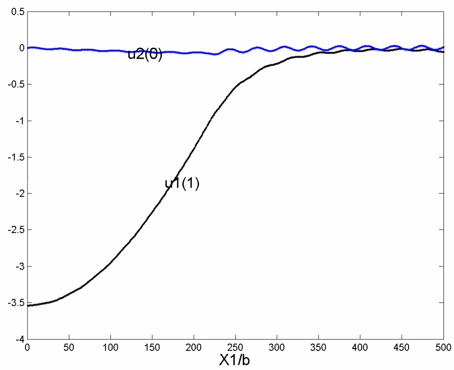

Figure 4. Calculated displacement of $u_2^{(0)}, u_1^{(1)}$ in X_1 direction to the frequency at point (1) in Fig. 3.

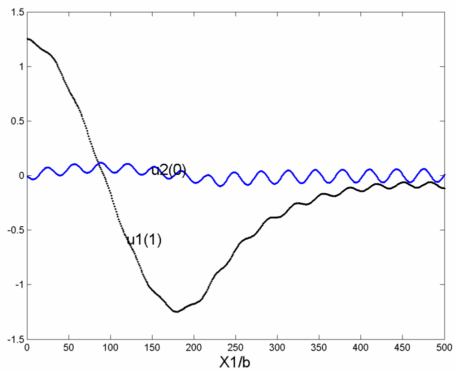

Figure 5. Calculated displacement of $u_2^{(0)}, u_1^{(1)}$ in X_1 direction to the frequency at point (2) in Fig. 3.

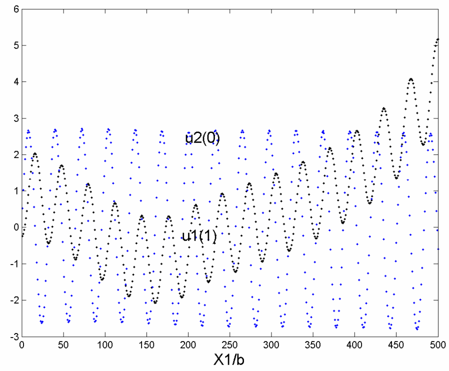

Figure 6. Calculated displacement of $u_2^{(0)}, u_1^{(1)}$ in X_1 direction to the frequency at point (3) in Fig. 3.

Table 1. Measured resonances of partially plated AT-cut crystal plate with different widths

Sample	*Ratio of width vs thickness*	*Normalized frequency of thickness shear*	*Normalized frequency of first inharmonic overtone*
I	31	0.967667	0.986593
II	32	0.967418	0.986071
III	33	0.967539	0.986313
IV	34	0.967545	0.986472
V	35	0.967303	0.986108

4. CONCLUSION

By examination of Mindlin first order equations for partially plated crystal plate, we obtain the frequency spectra and displacements. Compared the frequency spectra and measurement data, we find theoretical calculation still matches quite well for miniaturization quartz crystal resonators. The Relationship between length of electrode and first inharmonic overtone has been gotten. Both the measurement data and theoretical computation, proves that the length and width of quartz plate and the width of electrode have little influence of inharmonic overtone.

Turn to design improvement of crystal resonator, the conclusion is if spurious problems are accused by inharmonic overtone, changing the dimensions of length of electrode will be an effective method.

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