

THE EFFECT OF ELECTRODES ON HIGHER-ORDER OVERTONE VIBRATIONS OF THE THICKNESS-SHEAR MODE OF QUARTZ CRYSTAL PLATES

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The effect of metal electrodes on high frequency vibrations of plates has to be carefully considered in the analysis and design of quartz crystal resonators. Extensive studies have been done in the analysis of various electrode configurations, including the symmetric, asymmetric, and layered structures to predict the exact frequency and mode shapes which are important in the estimation of resonator properties and the optimal design parameters for the fundamental thickness-shear vibration mode. Through practical experiences, it is evident that the electrode structure has much stronger effects on the overtone vibrations of quartz crystal plates and resonators, but analytical solutions based on a simple physical resonator model have not been examined in earlier studies. In this paper, we start from the usual ideal model of an AT-cut quartz crystal plate covered by metal electrodes, and frequency equations, which have been obtained in earlier studies with and without the consideration of piezoelectric effect, are evaluated with known parameters to observe the changes of the vibration frequencies of different overtone modes and deformation. From calculation, it is clear that the frequency change is much large for overtone modes in comparison to the familiar fundamental vibrations of most quartz crystal resonators. The formulation can also be extended to consider the thermal effect of electrodes for the study of frequency-temperature relation of higher-order overtone modes. It is expected that by solving the familiar equations for frequency under the complication of electrodes at higher-order overtone frequencies, we should be able to refine and establish a procedure for the analysis and design which has been overlooked due to the complexity of the problem and lack of demands, which is now being promoted by the elevated frequency and miniaturization of resonators related to energy consumption reduction.

Keywords: Plate, vibration, quartz, crystal, frequency, piezoelectric, resonator, overtone, thickness-shear

1. Introduction

In applications, quartz crystal resonators are also designed and made to work at of overtone modes to acquire much higher frequencies in comparison to the usually lower frequency of the fundamental thickness-shear mode. Since overtone modes are with the same family of the fundamental mode, the deformation and frequencies can be determined with equations from the same theory of high frequency vibrations of crystal plates. As a result, the deformation and frequency can be evaluated with the same equations except different parameters based on the vibration mode. Consequently, the design procedure of overtone resonators is also similar to the one for the fundamental mode type. In order to have practical design tools and guidelines for overtone type resonators, we have to extend the frequency range of earlier studies on thickness-shear vibrations of quartz crystal plates to overtone frequencies which are odd multiples of the fundamental frequency. Current overtone resonators can be made as

high as to the 11th-order, but theoretical studies on vibrations and frequency of these overtone modes are rare. Particularly, in order to have good performance at these higher-order overtone modes, we need to study the effect of electrodes and mountings as essential structural complications on the vibration frequency of the particular mode and electrical parameters such as the capacitance ratio and impedance. Such analysis can be done with three-dimensional equations of infinite piezoelectric crystal plates [1-4] from the beginning, as we have been accustomed to in the analysis and design of the fundamental type quartz crystal resonators. Of course, for an accurate analysis, we need to use the two-dimensional plate theories, such as the Mindlin plate theory [5] and Lee plate theory [6-7], which are suitable for the analysis and design quartz crystal resonators.

As our first step in studying the structure complication and design consideration of overtone quartz crystal resonators in a systematic manner, we start with the effect of electrodes on the higher-order overtone

thickness-shear frequencies, which are one of essential device properties to be realized first. As our earlier work on the effect of electrodes on piezoelectric crystal plates [8], we use an infinite plate covered by symmetric electrodes to derive the frequency equation. The frequency equations without and with piezoelectric effect considerations can be solved for overtone frequencies of thickness-shear vibrations as for the fundamental mode. These frequencies, of course, equations can also be extended to asymmetrically electroded plates as studied by Yang et al [9].

After obtaining the frequency equation, we solve the transcendental equation with assumed electrode configurations to evaluate the effect on the frequency. The results show that for typical metal electrodes, frequency shifts caused by the mass and stiffness effects of electrodes increase with the order of vibration overtones, or it is larger for overtone modes. In other words, overtone vibration frequencies are more sensitive to electrodes. Since frequency equations from infinite plates are valid for all overtone modes, we can use these equations to determine the optimal electrode structure as part of the optimal design scheme [10-12]. Eventually, calculated results from this study will be used with measurements from actual products to establish proper design guidelines for overtone thickness-shear crystal resonators.

2. Vibration Frequency of the Thickness-shear Mode for An Electroded Crystal Plate

Since overtone modes are members of the thickness-shear vibration family of plates, we start from the general formulation of the thickness-shear vibrations of infinite plates for the frequency calculation. We use an electroded infinite plate of quartz crystal shown in Fig. 1 for our formulation. From Wang and Shen [8], thickness-shear displacements in the crystal plate and electrodes without the time factor are

$$\begin{aligned} u_1 &= A \sin \eta x_2, -b \leq x_2 \leq b, \\ \bar{u}_1 &= \pm A \sin \eta b \cos \bar{\eta} (x_2 \mp b) + \bar{B} \sin \bar{\eta} (x_2 \mp b), \\ & b \leq |x_2| \leq b + 2\bar{b}, \end{aligned} \quad (1)$$

where $A(B)$, $\eta(\bar{\eta})$, $b(\bar{b})$, and x_2 are vibration amplitudes, wavenumbers, thickness, and coordinate, respectively. The strain and stress components can be obtained from (1) for the isotropic electrodes and anisotropic blank. The general solutions of the vibration can be obtained from equations of motion [8].

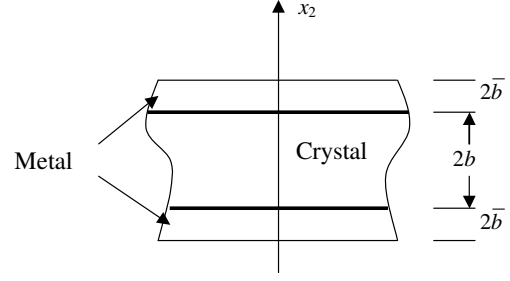


Fig.1 Electroded crystal plate of a typical crystal resonator

Applying the continuity and free boundary conditions on the interfaces and surfaces, we have the frequency equation of the electroded crystal plate as

$$\begin{aligned} \tan \xi \tan \frac{2B}{k} \xi &= \frac{k}{C_{66}}, \\ \eta &= k\bar{\eta}, k^2 = \frac{\bar{v}_2^2}{v_2^2}, v_2^2 = \frac{c_{66}}{\rho}, \bar{v}_2^2 = \frac{\bar{c}_{66}}{\bar{\rho}}, \\ B &= \frac{\bar{b}}{b}, C_{66} = \frac{\bar{c}_{66}}{c_{66}}, \xi = \eta b, \end{aligned} \quad (2)$$

where $\rho(\bar{\rho})$ and $c_{66}(\bar{c}_{66})$ are densities and elastic constants of the plate (electrode), respectively. By solving for ξ , we can have accurate prediction of the thickness-shear frequency solution.

It should be reminded that the frequency equation in (2) can be extended further to include the piezoelectric effect of the crystal blank as shown by Wang and Shen [8]. As our investigation on overtone modes, we start with the frequency equation given in (2).

It is generally known that when the plate is not electroded, the frequency solution for the fundamental thickness-shear vibration mode from (2) is $\xi = \pi/2$. For n th-order overtone modes, exact frequency solutions of the thickness-shear modes are $\xi = n\pi/2$. With the presence of electrodes on surfaces, the frequency will be slightly reduced, but exact solutions should be sought in the vicinity of the solution without electrodes. With electrode material and thickness known, we should be able to solve the frequency equation to see shifts of overtone modes.

As the major objective of this study, we also want to expand the frequency range of the equation to overtones of the thickness-shear mode so the design can be done more accurately for these high frequency resonators. In addition, we can use the relation between the frequency

and resonator structural parameters for the optimal design of each overtone product.

3. Numerical Examples

To show the effect of electrodes and crystal blanks on the frequency of overtone thickness-shear modes, we use the frequency equation to investigate frequency changes. The relationship between the frequency and symmetric electrodes will help the proper selection of electrodes based on the manufacturing process. The precise frequency and electrode information will be used to pinpoint the electrode processing and reduce the processing time and cost.

We use AT-cut quartz crystal as the blank and gold and silver as electrodes in the numerical calculation. For the electrodes, we define the familiar mass ratio as

$$R = \frac{2\bar{\rho}b}{\rho b} \quad (3)$$

For typical resonators, we always want to use small mass ratio R so the frequency shift can be minimized. The overtone frequency solutions are obtained from the frequency equation given in (2) in a manner similar for the fundamental frequency. For comparison purpose, shifts from the exact overtone frequencies of crystal plates without electrodes ($n\pi/2$) normalized to 1 are calculated for different electrodes.

For gold electrodes, frequency shifts for n th overtone modes of thickness-shear vibrations ($n = 1, 3, 5, 7,$ and 11) versus mass ratio are plotted in Fig. 2. For silver electrodes, the effects are plotted in Fig. 3. The frequency shifts versus mass ratio are plotted in Figs. 4 and 5, respectively.

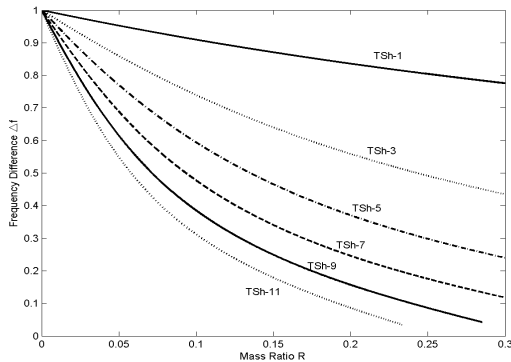


Fig. 2 Overtone thickness-shear frequency shifts from the unelectroded quartz crystal plate vs. the mass ratio R of gold electrodes.

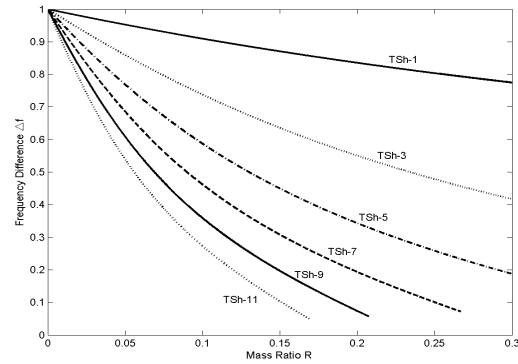


Fig. 3 Overtone thickness-shear frequency shifts from the unelectroded quartz crystal plate vs. the mass ratio of silver electrodes.

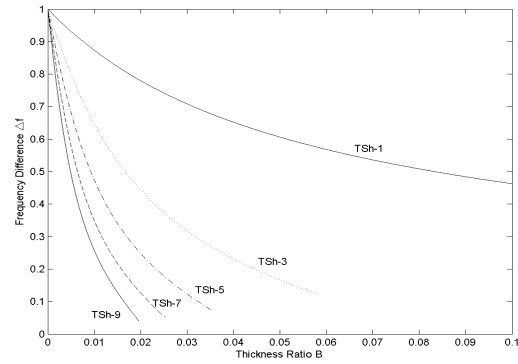


Fig. 4 Overtone thickness-shear frequency shifts from the unelectroded quartz crystal plate vs. the thickness of gold electrodes.

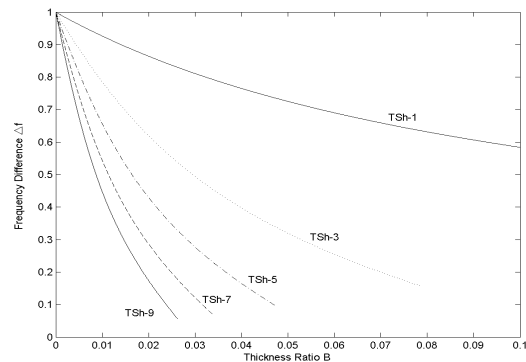


Fig. 5 Overtone thickness-shear frequency shifts from the unelectroded quartz crystal plate vs. the thickness of silver electrodes.

4. Conclusions

By solving frequency equations of thickness-shear vibrations of electroded quartz crystal plates at higher-order overtones of the thickness-shear mode, the frequency effect of symmetrically plated electrodes is studied. It is clear, as we can see from the frequency equation, the effect on the vibration frequency differs from the order of vibration modes, and large frequency changes are expected for the higher-order overtones. Although this is straightforward from the equation and experiences, the calculation based on the assumed quartz crystal blank and electrode information will be helpful in the design process of overtone type resonators as the proper selection of electrode material and thickness has to be incorporated. Obviously, the frequency equation can be easily rewritten so the design process can be reversed by determining the structural parameters with given frequency. Since essential parameters like the electrodes are known with manufacturing process, we can calculate the crystal blank thickness as a start. The simple equation and corresponding design process will certainly give the design process a firm theoretical foundation.

It should be reminded that the frequency equation given in this study is based on the fact that the plate is infinite. For frequency calculation as a starting point of the design process, this is adequate because the thickness-shear frequency change due to the length of blank is really limited if the blank is longer than a few thickness. Actually, the fundamental thickness-shear frequency is determined based on this fact also. The consideration of the crystal blank and electrode dimensions is important for more essential parameters of the overtone resonators should be studied with plate theories like Mindlin and Lee in our subsequent work of this project. We can combine the exact frequency solutions and advantage of the two-dimensional plate theory for the further analysis of the overtone quartz crystal resonators.

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